(a) : Given $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}$ and $z \in C$

$$\Rightarrow \left|\frac{\bar{z}-i}{\bar{z}+\frac{i}{2}}\right|=\frac{2}{3} \Rightarrow 3|x-iy-i|=2|x-iy+\frac{i}{2}|$$

$$\Rightarrow 3|x-i(y+1)|=2\left|x-i\left(y-\frac{1}{2}\right)\right|$$

$$\Rightarrow 3\sqrt{x^2+(y+1)^2}=2\sqrt{(x)^2+\left(y-\frac{1}{2}\right)^2}$$

Squaring on both sides, we get

$$9(x^2+y^2+1+2y)=4\left(x^2+y^2+\frac{1}{4}-y\right)$$

$$\Rightarrow 9x^2+9y^2+9+18y=4x^2+4y^2+1-4y$$

$$\Rightarrow 5x^2+5y^2+22y+8=0$$

$$\Rightarrow x^2+y^2+\frac{22}{5}y+\frac{8}{5}=0$$

Centre (C) $=\left(0,-\frac{11}{5}\right)$

Now, $\frac{1}{2}\left|0+0+\alpha\left(0+\frac{11}{5}\right)\right|=11$ [Given]

$$\Rightarrow \frac{11\alpha}{5}=22 \Rightarrow \alpha=10 \quad \therefore \alpha^2=100$$